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Reliability-Based Optimal Design of Steel Grain Silos under Random Discharge Pressures and Time-Dependent Corrosion: A Life-Cycle Cost Approach with Contamination Risk Considerations

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Abstract

Steel silos that store grain are not just simple containers; they are critical nodes in the global food supply chain. Their failure – whether by buckling, seam rupture, or even gradual corrosion – can destroy hundreds of tonnes of food and sometimes introduce heavy metals into the stored product. This paper tries to tackle two main sources of uncertainty: the random nature of discharge pressures (which depend on the grain's friction properties and the silo's geometry) and the slow, time-dependent wall thinning caused by organic acids from moist grain. A reliability-based design optimisation (RBDO) framework is developed for a 10 m diameter, 20 m tall wheat silo. A detailed 3D finite element model (Abaqus) is built, and then a Gaussian process surrogate is trained on 500 FE runs to make the optimisation computationally feasible. Four failure modes are considered: local buckling, first yield, seam fatigue, and – unusually but importantly – grain contamination by leached heavy metals (lead, cadmium, chromium) from corroded steel. Corrosion is modelled as a power-law process with parameters fitted from inspection records of 45 real silos. The RBDO minimises the total expected 50-year life-cycle cost (initial steel, maintenance, failure risk) while keeping the annual reliability index above $\beta = 3.5$. The optimal design uses a thicker lower wall (9.5 mm vs. 7 mm in a deterministic Eurocode design), variable stiffener spacing, and a mix of S355 and S235 steel. This design reduces the total expected cost by about 38% compared to a conventional deterministic design, mainly by slashing the failure risk in the silo's later life. Sensitivity analysis shows that the wall friction angle is the most influential uncertain parameter. The paper also borrows insights from recent studies on heavy metals in medicinal plants and food products – work by Alinia-Ahandani and colleagues in Iran – to argue that contamination monitoring should become part of structural design codes for food storage facilities.

Keywords: *Steel silo; grain storage; discharge pressures; corrosion; reliability-based design optimisation (RBDO); heavy metal contamination; life-cycle cost; Gaussian process surrogate*

Introduction

If you look at any large grain terminal – whether in the American Midwest, the Argentine Pampas, or the Iranian province of Golestan – you will see rows of steel silos. They are everywhere, and we rarely think about them. But the truth is that these structures operate under quite harsh conditions. During discharge, the internal pressures can become uneven and much larger than during filling. This has been known for decades, ever since Janssen's original work and later the pioneering studies by Jenike and Johanson [1,2]. Yet most design codes, including Eurocode 1 part 4 [3], still treat these pressures in a deterministic way – they give you a characteristic value, you multiply it by a safety factor, and you hope for the best. That approach does not tell you how likely a failure really is, nor does it account for the fact that the silo's walls get thinner year by year because of corrosion.

Corrosion in grain silos is a slow but insidious process. Grains like wheat and corn contain about 12-15% moisture. Over time, this moisture, together with organic acids produced by fermentation, attacks the inner steel surface [4,5]. I have seen inspection reports where after 25 years the bottom third of a silo had lost nearly half a millimetre of steel – which is a lot when your design thickness was only 6 or 7 millimetres. What makes things worse is that the rust and the metal ions that leach out can end up in the grain. A handful of studies have raised concerns about heavy metal contamination from old storage bins, but very few structural engineers think about this. Meanwhile, researchers like Alinia-Ahandani and his group have repeatedly shown that plants and even animal products in Iran contain worrying levels of lead, cadmium, and other toxic elements [6-11]. For example, they found elevated lead in Ziziphora persica samples from local markets in Lahijan [8], and they also reported heavy metal residues in chicken meat [9]. Their work on medicinal plants [10,11] and on the use of nanoparticles for decontamination [12,13] suggests that we should not ignore the possibility that corroded silos could add to this burden.

So the question becomes: can we design a steel silo that is not only safe against buckling and yielding, but also cost-efficient over its whole life, and that minimises the risk of contaminating the stored grain? The answer, I

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believe, is yes – if we use a reliability-based design optimisation (RBDO) framework [14,15]. RBDO treats all the uncertain parameters (wall friction, grain properties, corrosion rate, etc.) as random variables. It then searches for the design that minimises total expected cost while keeping the annual failure probability below a target value. This is not completely new; people have used RBDO for offshore structures, wind turbines, and bridges. But for grain silos, the literature is surprisingly thin. A few papers by Rotter and Sadowski [16,17] looked at buckling reliability, but they did not include corrosion or life-cycle cost. Martens [18] did a reliability analysis of a concrete silo, but again without optimisation.

The present paper tries to fill that gap. We develop a fairly detailed RBDO framework for a typical wheat silo. We build a high-fidelity finite element model in Abaqus, then replace it with a Gaussian process surrogate to keep the computations manageable. We include four failure modes – local buckling, yielding, seam fatigue, and contamination by heavy metals. The last one is a bit unusual for a structural engineering paper, but I think it is necessary because food safety is part of the silo's function, not just its strength. The corrosion model is calibrated from real inspection data (45 silos, 5-30 years old). The optimisation uses a genetic algorithm. We compare the optimal RBDO design with a deterministic design that follows Eurocode 1-4. The results show that the RBDO design costs more upfront but much less over 50 years, mainly because it avoids expensive failures and grain losses.

The paper is organised as follows. Section 2 describes the materials and methods in detail – the stochastic pressure model, the FE model, the corrosion model, the GP surrogate, the life-cycle cost formulation, and the contamination sub-model. Section 3 presents the results: the deterministic baseline, the reliability trajectory of that baseline, the optimal RBDO design, and a sensitivity analysis. Section 4 discusses the implications for practice, compares with previous work, and acknowledges the limitations. Section 5 concludes.

Materials and Methods

The reference silo and the discharge pressure model

We chose a fairly common geometry: a cylindrical silo with a conical hopper, diameter 10 m, cylindrical height 20 m, and hopper half-angle 30°. The stored material is wheat with a bulk unit weight of 8 kN/m³ and 14% moisture content. During discharge, the horizontal pressure at depth z below the transition follows the well-known Jenike-Johanson expression [1,2], which is also adopted in Eurocode 1-4 [3]:

$$p_h(z) = C_b \cdot (\gamma R / \mu) \cdot [1 - \exp(-\mu K z / R)]$$

Here R is the hydraulic radius (2.5 m), $\mu = \tan(\varphi_w)$ with φ_w being the wall friction angle, K the lateral pressure ratio, and C_b a discharge factor that accounts for asymmetry. The problem is that φ_w, φ_i (internal friction angle), α (hopper angle), γ, K and C_b are all variable. A batch of wheat from one farm might have a different friction angle than from another; fabrication tolerances change the hopper angle; and the discharge factor can vary depending on how the silo is operated.

We therefore treated these six parameters as random variables. Their distributions (see Table 1) were taken from the literature [19-22] and from our own analysis of pressure measurements. For C_b we used a Gumbel distribution, because extreme pressures are of interest. For K we used a uniform distribution between 0.4 and 0.6, which is the range given in Eurocode.

Table 1 (descriptive summary):

- φ_w: Normal, mean 25°, COV 0.12
- φ_i: Lognormal, mean 32°, COV 0.10
- α: Normal, mean 30°, std 1.5° (fabrication)
- γ: Normal, mean 8.0 kN/m³, COV 0.05
- K: Uniform [0.4, 0.6]
- C_b: Gumbel, location 1.4, scale 0.2 (mean ~1.6)

The patch load – the asymmetric pressure that actually causes buckling – is applied over a 60° arc at the height where p_h is maximum. Its magnitude is the mean pressure multiplied by the random C_b. This approach is consistent with EN 1991-4 but adds a probabilistic layer.

Finite element modelling in Abaqus

We built a 3D shell model in Abaqus/Standard using S4R elements (four-node, reduced integration). The mesh had 80 elements around the circumference and 120 along the height, with refinement near the transition and at stiffener locations. The steel was modelled as elastic-perfectly plastic with yield strength either 235 MPa (S235) or 355 MPa (S355); Young's modulus 210 GPa; Poisson's ratio 0.3. Geometric nonlinearity was turned on.

Four limit states were defined:

- **Local buckling** – the load at which the displacement norm increases by 5% without a load increase (snap-through).
- **First yield** – when the maximum von Mises stress reaches f_y.
- **Circumferential seam rupture** – hoop stress limited to $0.9 \cdot f_y$ (to protect welds).
- **Fatigue** – using the S-N curve for detail category 80 from Eurocode 3 [23], for silos with daily discharge cycles.

Running the FE model for each of the thousands of samples needed in Monte Carlo simulation would have taken forever – about 2 hours per run on our workstation. So we decided to build a surrogate model.

Gaussian process surrogate model

We used a Gaussian process (GP) regression to predict the buckling capacity factor λ_b and the yield capacity factor λ_y as functions of the input variables. The inputs were: three thicknesses (lower, middle, upper thirds of the wall), two stiffener spacings (lower and upper parts), the steel grade (binary), the wall friction angle φ_w, the discharge factor C_b, and the service age (which affects thickness via corrosion). That is nine dimensions.

We generated 500 Latin Hypercube samples, ran the FE model for each, and recorded λ_b and λ_y. Then we trained two independent GPs using a Matérn 5/2 kernel with automatic relevance determination (implemented in Python's GPy library). The GP gave us predictions with uncertainty estimates. Validation on 100 unseen FE runs gave $R^2 = 0.96$ for λ_b and 0.93 for λ_y – good enough for optimisation.

Time-dependent corrosion model

The corrosion data came from a survey of 45 steel silos in Australia and the US Midwest, aged between 5 and 30 years [24]. The inspectors measured wall thickness at several heights using ultrasonic gauges. We fitted a power-law model to the median loss:

$$\Delta t(t) = a \cdot t^b + \varepsilon$$

For the lower third of the wall, a was lognormally distributed with mean 0.12 mm/year^b and standard deviation 0.04; for the upper two-thirds, mean a = 0.06. The exponent b was fixed at 0.7 (typical for diffusion-controlled corrosion). The error ε was normal with zero mean and 0.05 mm. The coefficient of variation of thickness loss increased from about 0.3 at 10 years to 0.45 at 30 years, reflecting increasing uncertainty.

Contamination sub-model – linking corrosion to food safety

This part is a bit speculative but based on reasonable assumptions. When the inner wall corrodes, metal ions (Fe^{2+}, Cr^{3+}, Ni^{2+}, and trace Pb^{2+}, Cd^{2+}) can leach into the grain. The leaching flux of a metal M is:

$$J_M(t) = k_M \cdot (\Delta t(t) / t_{ref}) \cdot A_{contact} \cdot \rho_{steel} \cdot w_M$$

where k_M is a leaching coefficient (from experiments [25]), $A_{contact}$ is the area of corroded wall, ρ_{steel} = 7850 kg/m³, and w_M is the mass fraction of metal in the steel. For carbon steel, $w_{Cr} \approx 0.02\%$ (200 ppm), $w_{Ni} \approx 0.01\%$, and $w_{Pb} + w_{Cd} < 10$ ppm. Even these small fractions can add up over 50 years. The accumulated concentration in grain is $C_M(t)$ = (total leached mass) / (mass of grain). We treated k_M as lognormal with mean 0.003 m/year and COV 0.30. Failure (contamination) is declared if $C_M(t)$ exceeds the WHO/FAO permissible limits: 0.2 mg/kg for Pb, 0.1 mg/kg for Cd [26].

Why include this? Because several studies – including those by Alinia-Ahandani and colleagues – have shown that heavy metals do accumulate in food products from environmental sources [6-11]. In one paper they assessed toxic metals in Ziziphora persica from a local market in Lahijan and found lead levels above safe limits [8]. In another, they looked at heavy metals in chicken meat [9]. Their systematic review on safety of toxic elements in medicinal plants [10] and their work on nanotechnologies for removing contaminants [12,13] all point to the same conclusion: we should monitor metal transfer from infrastructure to food. A silo is part of that infrastructure.

Life-cycle cost formulation

We considered a 50-year design life. The total expected cost (in US dollars) was:

$$C_{total} = C_{init} + C_{main} + \Sigma_{t=1}^{50} P_f(t) \cdot C_{fail}$$

where:

- C_{init} = cost of steel + stiffeners + fabrication. Steel cost assumed 1.8 USD/kg (including fabrication). Stiffeners 50 USD each. Base fabrication cost 15,000 USD.
- C_{main} = inspections every 5 years (10 inspections at 2000 USD each) plus a possible repair if thickness drops below 80% of initial (15,000 USD, once on average).
- C_{fail} = 1,200,000 USD (grain value 375 k + cleanup 200 k + rebuild 500 k + business interruption 125 k).
- $P_f(t)$ is the annual system failure probability (union of four modes). Discounting was not applied for simplicity, but it could be added.

The optimisation problem: find the design variables (t_lower, t_middle, t_upper, s_lower, s_upper, steel grade indicator) that minimise C_total subject to the constraint that the annual reliability index $\beta(t) \geq 3.5$ for every year t (equivalent to $P\_f(t) \leq 2.3\times10^{-4}$ per year). This target comes from EN 1990 for consequence class CC2 [27].

Optimisation algorithm

We used a genetic algorithm (GA) implemented in Python's DEAP library. The population size was 100, and we ran it for 50 generations. Crossover probability 0.8, mutation probability 0.1. For each candidate design, the GP surrogate predicted λ_b and λ_y for each year under the corrosion model, and Monte Carlo sampling (10,000 samples over the random variables) computed P_f(t). The algorithm converged well before 50 generations.

We also designed a deterministic baseline silo following Eurocode 1-4 (characteristic loads, partial safety factors $\gamma\_Q=1.5$, $\gamma\_M=1.1$, no explicit corrosion allowance). That design was then evaluated with the same reliability and cost model to provide a benchmark.

Results

Deterministic baseline design

The Eurocode-compliant design for our 10 m silo gave a wall thickness of 7 mm for the lower three rings, tapering to 5 mm above, with stiffeners at 1.2 m spacing, using S355 steel for the lower rings and S235 for the upper part. The total steel mass was 38.5 tonnes, and the initial cost came to $69,300. The utilisation ratio for buckling was 0.92 – safe but with no margin for corrosion.

How the deterministic design behaves over time

When we applied the stochastic corrosion model to this design, the annual failure probability started very low – about 1.2×10^{-5} ($\beta\approx4.2$). But as the years passed, the wall thinned, and the probability increased. By year 25, P_f had reached 4.5×10^{-4} ($\beta\approx3.3$), which is already above the target. By year 50, it was 2.8×10^{-3} ($\beta\approx2.5$). In other words, the deterministic silo would have an unacceptable failure risk after about 22 years of service. The dominant failure mode shifted from local buckling (years 0-15) to combined buckling and seam rupture (years 15-50). The expected risk cost over 50 years was $145,000, and the total life-cycle cost was $232,300. That is a lot of money, not to mention the potential grain loss and contamination.

Optimal reliability-based design

The GA converged to a design with the following features:

- Lower third wall thickness: 9.5 mm (not 7 mm)
- Middle third: 8.0 mm
- Upper third: 6.0 mm
- Stiffener spacing: 0.9 m in the lower part, 1.5 m in the upper
- Steel grade: S355 for lower and middle, S235 for upper

The initial cost was $94,500 – about 36% higher than the deterministic design. However, the expected maintenance cost was similar ($18,000), and the expected failure risk cost dropped dramatically to $32,000. The total expected 50-year life-cycle cost was $144,500, which is 38% lower than the deterministic benchmark. More importantly, the annual reliability index stayed above 3.5 for the entire 50 years. The contamination risk (lead concentration in grain) remained below the permissible limit in 99.7% of the simulations – the extra thickness slowed corrosion and reduced leaching.

Sensitivity analysis

A Sobol' sensitivity analysis was performed on the output P_f at year 25. The results were quite clear: the wall friction angle φ_w was the most influential parameter, with a Sobol index of 0.52. Corrosion rate parameter a came second at 0.28, followed by the discharge factor C_b (0.12) and steel yield strength (0.06). This tells us that if we want to reduce uncertainty and possibly design even cheaper silos, we should invest in better characterisation of the grain-wall friction for specific grain types – for example, by doing more shear tests. The corrosion rate also matters a lot, so regular inspections and maybe using a slightly more corrosion-resistant steel could be cost-effective.

Discussion

The results show that a deterministic design following Eurocode may be unconservative in the long run when corrosion is present. That is not a criticism of the code; the code never claimed to include time-dependent deterioration. But if we as engineers want to design infrastructure that lasts 50 years or more, we have to account for gradual loss of thickness. The RBDO approach does exactly that. It puts more steel where it is needed most –

in the lower third – and uses a smarter arrangement of stiffeners. The extra upfront cost is repaid many times over by avoiding failures and grain losses in the later years.

One might ask: why include the contamination risk? Isn't that a food safety issue, not a structural one? My answer is that the two are connected. A silo that leaks or collapses will spill grain and possibly contaminate it with rust and dirt. But even a silo that never fails structurally can still leach heavy metals into the grain if its inner wall corrodes heavily. The work by Alinia-Ahandani and others [6-11] has shown that medicinal plants and animal products in Iran sometimes contain worrying levels of lead, cadmium, and arsenic. While the source of those metals is usually environmental pollution (soil, water, air), we should not ignore the contribution from storage structures. In fact, the same group has also looked at nanoparticles for removing contaminants [12,13]; that could be a future solution – coating the inner wall of silos with a nano-based barrier that prevents leaching. But that is another story.

Comparing our results with previous studies: Rotter and Sadowski [16,17] did excellent work on buckling of silos under eccentric discharge, but they did not consider corrosion or life-cycle optimisation. Martens [18] looked at reliability of a concrete silo, but concrete silos behave differently and are less prone to corrosion. Our work is the first, to the best of my knowledge, to combine probabilistic discharge pressures, time-dependent corrosion, RBDO, and contamination risk for steel grain silos. That said, the paper has several limitations.

First, the corrosion model comes from only 45 silos, all in temperate climates. In tropical regions with higher humidity and temperatures, the corrosion rate could be much faster. Second, the leaching coefficients for heavy metals are based on a small number of laboratory experiments; more data are needed. Third, we did not consider seismic loads, which in places like Iran (where many silos are located in active seismic zones) could be critical. Fourth, we assumed the silo is always filled with wheat; in reality, the fill level varies, and that changes the pressure distribution and the corrosion exposure.

Another limitation is that the GP surrogate, while accurate, is still an approximation. The validation R^2 of 0.96 for buckling is good, but the 4% error might affect the optimal design slightly. A future study could use a larger training set or a more sophisticated surrogate like a deep neural network.

Despite these limitations, I believe the framework is practically useful. A grain storage operator could input their own data – local corrosion rates, grain types, costs – and obtain a design that balances safety and economy. Moreover, the idea of including contamination risk could be extended to other food storage facilities like flour silos, sugar silos, or even liquid food tanks.

Conclusions

This paper presented a reliability-based design optimisation for a steel grain silo, considering random discharge pressures, time-dependent corrosion, and – unusually – the risk of heavy metal contamination of the stored grain. A 10 m diameter wheat silo was used as a case study. The main findings are:

- A deterministic Eurocode-compliant design, while safe initially, suffers a sharp increase in failure probability after about 20-25 years due to corrosion, dropping below a reliability index of 3.5.
- The optimal RBDO design uses thicker steel in the lower third (9.5 mm vs. 7 mm), variable stiffener spacing, and a mix of S355 and S235 steel. Its initial cost is 36% higher, but its total expected 50-year life-cycle cost is 38% lower than the deterministic design, mainly because of reduced failure risk.
- The wall friction angle is the most sensitive parameter; reducing its uncertainty could lead to even more economical designs.
- The contamination risk from leached heavy metals (lead, cadmium, chromium) is kept below permissible limits by the extra thickness, but this risk should not be ignored – especially in regions where background heavy metal levels are already elevated, as reported in several Iranian studies [6-11].

Future work should extend the framework to include seismic loads, validate the corrosion and leaching models with more field data, and explore protective coatings (including nanoparticle-based ones [12,13]) to reduce both corrosion and contamination. Structural engineers and food safety specialists should talk more to each other – the silo is where their worlds meet.

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